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06CS34

Third Semester B.E. Degree Examination, Dec. 2013/Jan. 2014
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. For any three sets A, B, C prove that
$(A - B) - C = A - (B \cup C) = (A - C) - (B - C)$. **(06 Marks)**

b. A student visits an arcade each day from Monday to Friday after school and plays one game of either loser man, millipede, or space conquerors. In how many ways can he play one game each day so that he plays each of the three at least once during a week (from Monday to Friday)? **(07 Marks)**

c. A girl rolls a fair die three times. What is the probability that i) her second and third rolls are both larger than her first roll ii) the result of her second roll is greater than that of the first roll and the result of her third roll is greater than that of the second? **(07 Marks)**

2 a. Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following :
i) $p \wedge q$ ii) $\neg p \vee q$ iii) $q \to p$ iv) $\neg q \to \neg p$ v) $\neg p \leftrightarrow \neg q$. **(05 Marks)**

b. Define tautology. Prove that, for any propositions p, q, r, the compound proposition
$\{p \to (q \to r)\} \to \{(p \to q) \to (p \to r)\}$ is a tautology. **(05 Marks)**

c. Without using truth tables, prove the following logical equivalence :
$(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(05 Marks)**

d. Test the validity of the following argument :
If I study, I will not fail in the examination
If I do not watch TV in the evenings, I will study
I failed in the examination
∴ I must have watched TV in the evenings. **(05 Marks)**

3 a. Consider the following open statements with the set of all real numbers as the universe.
$p(x) : x \geq 0$, $q(x) \geq 0$
$r(x) : x^2 - 3x - 4 = 0$
determine the truth values of the following statements
i) $\exists x,\ p(x) \wedge q(x)$ ii) $\forall x,\ p(x) \to q(x)$ iii) $\forall x,\ r(x) \to p(x)$. **(06 Marks)**

b. Prove that the following argument is valid :
$\forall x, [p(x) \to \{q(x) \wedge r(x)\}]$
$\underline{\forall x, [p(x) \wedge s(x)]}$
$\therefore \forall x, [r(x) \wedge s(x)]$. **(07 Marks)**

c. Give : i) a direct proof ii) an indirect proof iii) proof by contradiction, for the following statement : "If n is an odd integer, then n + 11 is an even integer". **(07 Marks)**

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(06 Marks)**

b. Find an explicit definition of the sequence defined recursively by
$a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(07 Marks)**

c. State the pigeonhole principle. Prove that in any set of 29 person at least five persons must have been born on the same day of the week. **(07 Marks)**

PART – B

5 a. Define Cartesian product of two sets. For any three non – empty sets A, B, C prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(05 Marks)**

b. Let A = {1, 2, 3, 4} and let R be the relation on A defined by x R y if and only if "x divides y". Represent the relation R as a matrix and draw its diagraph. **(05 Marks)**

c. Let R = {(1, 2) (3, 4), (2, 2)} and S = {(4, 2), (2, 5) (3, 1) (1, 3)} be relations on the set A = {1, 2, 3, 4, 5}. Find the following i) R_0 (RoS) ii) S_0(SoR). **(05 Marks)**

d. Let A = {1, 2, 3, 4, 5}. Define a relation R on $A \times A$ by $(x_1, y_1)\ R\ (x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$

i) Verify that R is an equivalence relation on $A \times A$

ii) Determine the equivalence class [(1, 3)]. **(05 Marks)**

6 a. Define a partially ordered set. Let A = {1, 2, 3, 4, 6, 12} on A, define the relation by a Rb if and only if a divides b. prove that R is a partial order on A. draw the Hasse diagram for this relation. **(06 Marks)**

b. Define : i) one – one function ii) onto function.

Let A = {0, ± 1, ± 2, 3}. Consider the function $f : A \to R$ defined by $f(x) = x^3 - 2x^2 + 3x + 1$, for $x \in A$. Find the range of f. **(07 Marks)**

c. Let A = B = C = R, and $f : A \to B$ and $g : B \to C$ defined by $f(a) = 2a + 1$, $g(b) = \frac{1}{3}b$, $\forall a \in A$, $\forall b \in B$ compute g∘f and show that g∘f is invertible. What is $(g \circ f)^{-1}$. **(07 Marks)**

7 a. Define an Abelian group. Prove that a group G is abelian if and only if $(ab)^2 = a^2b^2$. **(06 Marks)**

b. Define a cyclic group. Prove that every cyclic group is abelian. **(07 Marks)**

c. Define left and right Cosets. State and prove Lagrange theorem. **(07 Marks)**

8 a. Define a ring. If R is a ring with unity and a, b are units in R, prove that ab is a unit in R and that $(a\,b)^{-1} = b^{-1}a^{-1}$. **(06 Marks)**

b. An encoding function $E : Z_2^2 \to Z_2^5$ is given by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

i) Determine all the code words

ii) Find the associate parity – check matrix H. **(07 Marks)**

c. Define a group code.

Consider the (6, 2) encoding function $E = Z_2^2 \to Z_2^6$ defined by E (00) = 000000, E(10) = 101010, E(01) = 010101, E (11) = 111111. Prove that $C = E(Z_2^2)$ is a group code. **(07 Marks)**

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